5. cvičení - výsledky

Příklad 1.

(a) $f'_x(u(x,y),v(x,y)) = 2e^{2x}\sqrt{1+e^{2x}} + \frac{e^{3x}}{2\sqrt{1+e^{2x}}}$
$f'_y(u(x,y),v(x,y)) = 0$

(b) $f'_x(u(x,y),v(x,y) = -\cos^3 x \cdot e^{3x} + 2\sin^2 x \cos x e^{3x} - 3\sin x \cos^2 x e^{3x}$
$f'_y(u(x,y),v(x,y)) = 0$

(c) $f'_x(u(x,y),v(x,y)) = 2(x-y)\cos\left((x-y)^2\right)\cos(x^2-y^2) - 2x\sin\left((x-y)^2\right)\sin\left(x^2-y^2\right)$
$f'_y(u(x,y,z),v(x,y,z)) = 2(y-x)\cos\left((x-y)^2\right)\cos(x^2-y^2) + 2y\sin\left((x-y)^2\right)\sin\left(x^2-y^2\right)$

(d) $f'_x(u(x,y),v(x,y) = -3e^{3x-2y} + \frac{xy+z}{x+2y+3z} + 2\log(x+2y+3z)\frac{y}{2}$
$f'_y(u(x,y,z),v(x,y,z)) = 3e^{3(x-y)} + \frac{2(xy+z)}{x+2y+3z} + x\log(x+2y+3z)$
$f'_z(u(x,y,z),v(x,y,z)) = \frac{3\sqrt{xy+z}}{x+2y+3z} + \log(x+2y+3z)$

Příklad 5.

(a) $\frac{\partial\varphi}{\partial x}(1,0) = 1$, $\frac{\partial\varphi}{\partial y}(1,0) = 0$, $\frac{\partial\psi}{\partial x}(1,0) = 0$, $\frac{\partial\psi}{\partial y}(1,0) = 1$.

(b) $\frac{\partial\varphi}{\partial x}(1,2) = 0$, $\frac{\partial\varphi}{\partial y}(1,2) = -\frac{1}{3}$, $\frac{\partial\psi}{\partial x}(1,2) = -1$, $\frac{\partial\psi}{\partial y}(1,2) = \frac{1}{3}$.

(c) $\frac{\partial\varphi}{\partial x}(1+e,e) = \frac{1}{e+1}$, $\frac{\partial\varphi}{\partial y}(1+e,e) = 0$, $\frac{\partial\psi}{\partial x}(1+e,e) = \frac{-e}{e+1}$, $\frac{\partial\psi}{\partial y}(1+e,e) = 1$.

(d) $\frac{\partial\varphi}{\partial x}(1,1) = -\frac{17}{22}$, $\frac{\partial\varphi}{\partial y}(1,1) = \frac{1}{22}$, $\frac{\partial\psi}{\partial x}(1,1) = \frac{-3}{22}$, $\frac{\partial\psi}{\partial y}(1,1) = -\frac{5}{22}$.

Příklad 6.

(a) $\varphi'(1) = -1, \varphi''(1) = 0$ a tečna je $x = 2 - y$.

(b) $\varphi'(-1) = -2, \varphi''(-1) = 2$ a tečna je $x = -2y$.

(c) $\varphi'(1) = \frac{1}{3}, \varphi''(1) = \frac{32}{27}$ a tečna je $x = \frac{y}{3} + 1 - \frac{\pi}{3}$.

Příklad 7.

(a) $u_x(1,1) = \frac{1}{2}, v_x(1,1) = \frac{-1}{2}$

(b) $u'(1,0) = \left(0, -\frac{1}{3}\right), v'(1,2) = \left(-1, \frac{1}{3}\right)$

(c) $z_{xx}(1,-1) = \frac{1}{2}, z_{xy}(1,-1) = \frac{1}{2}, z_{yy}(1,-1) = \frac{1}{2}$

(d) $z_{xy}(3,3) = \frac{26}{11}$

(e) $x_v(5,-7) = -\frac{1}{2}, y_v(5,-7) = \frac{1}{4}, z_v(5,-7) = \frac{2}{3}$

(f) $\frac{\partial\Phi}{\partial u}(0,1) = \frac{1}{e-2}$

Příklad 8.

(a) $\varphi'(\frac{\pi}{4}) = -1, \varphi''(\frac{\pi}{4}) = -\frac{32}{9}$ a tečna je $y = \frac{\pi}{2} - x$.

(b) $\varphi'(\pi) = \frac{2e^{\pi}-3}{2e^{\pi}+3}, \varphi''(\pi) = -24\frac{2e^{2\pi}-3}{(2e^{2\pi}+3)^3}$ a tečna je $y = \pi + \frac{2e^{\pi}-3}{2e^{\pi}+3}(x-\pi)$.

(c) $\varphi'(2) = -\frac{2}{5}, \varphi''(2) = \frac{24}{125}$ a tečna je $y = -1 - \frac{2}{5}(x-2)$.

(d) $\varphi'(1) = -\frac{1}{3}, \varphi''(1) = -\frac{70}{27}$ a tečna je $y = 1 - \frac{1}{3}(x-1)$.